Efficient secure AC OPF for distributed generation uptake maximisation

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Abstract - A method is presented for maximising the total capacity of distributed generation which a network can support, using a Security Constrained AC Optimal Power Flow (SCOPF). The motivation behind this model is to assess the network's capacity for new generation. Differently from the classical OPF, where generation cost is minimised, the objective function maximises the total capacity of the proposed DG sites that can be accommodated without breaching transmission constraints. Unlike previous work, a secure OPF including line outage contingencies is used, in order to ensure that the resulting DG capacities maintain the relevant network security standard. The resulting mathematical program is solved by iteratively adding the most severe contingencies and using warm starts to speed up the solution of the resulting OPFs. The results presented are based on a modified IEEE 73-bus Reliability Test System with N-1 security – however the method is generic and can be used at other voltage levels and with reconfiguration-based security models. Issues arising from the non-convexity of the AC OPF are also discussed.

Keywords - Optimal power flow, security constrained, generation planning, modelling language.

1 Introduction

WITH the current increase in renewable generation, the geographical pattern of generator locations is changing. Renewables must be located where the relevant resource is available, or where there is sufficient free land. As a result, there is now significant penetration of distributed generation (DG) into parts of the network, particularly the distribution network, where formerly there was mainly load. In order to maximise the potential of a network to support such distributed generation, careful planning is required - connection of generation in the wrong place can result in network sterilisation, significantly reducing the total capacity for DG [1].

Several authors have proposed mathematical optimisation-based approaches to network-wide planning of DG location, as opposed to choosing and sizing one DG site at a time. These have included linear programming [2] and genetic algorithm [3], as well as the use of a full AC optimal power flow to allocate DG capacity subject to network and fault level constraints [4].

The optimal power flow (OPF) was originally developed in the 1960s for network-constrained economic dispatch. With the immense increase in computing power since, OPF-based methods have been put to many uses [5]. The full AC OPF model is a non-convex, nonlinear optimization problem.

All transmission networks and many distribution networks are operated in a secure mode, to ensure continuity of supply under forced outage of a line (N-1 security), or in some cases any two lines (N-2 security). In order to obtain useful results from an OPF model, it is therefore often necessary to add security constraints. As this increases the size of the model significantly, pre-selection of a limited number of outage contingencies which are likely to be significant is frequently performed [5]. An alternative approach, particularly common in linearised models (where individual line flow constraints as opposed to entire contingency flows may be added) is to use an iterative method, at each step solving the secure OPF and adding to the optimisation problem any constraints violated by the resulting solution [6].

Once the contingencies have been chosen, most solution methods for the OPF mathematical program fall into three broad categories:

1. DC OPF models. The full nonlinear OPF model is linearised about zero voltage angle, all voltages are considered to be nominal, and reactive power flows and losses are neglected. While this allows for fast solution times, it is not realistic when reactive flows and voltage constraints are significant.

2. Sequential linear programming (SLP). The AC OPF is linearised about the current solution, the resulting linear program solved, and the OPF linearised about the new solution. This iteration continues until convergence to a solution of the original nonlinear problem (NLP) is achieved [7].

3. Direct solution of the AC OPF. The model is sent to a non-linear program solver. While SLP is a general NLP method, these are usually regarded in the OPF literature as distinct approaches, as the LP-based formulation provides opportunities to accelerate convergence, e.g. using a decoupled power flow.

In this paper, earlier work on network generation capacity assessment by use of an optimal power flow model to maximise generation [1, 4] is extended to include security constraints. Here, differently from the normal cost minimisation OPF, the new generator capacities are decision variables in the optimisation problem, and the objective function is the total DG capacity. The network constraints in the optimisation model ensure that it is possible to transmit the power generated to the nationwide transmission network.

This is demonstrated using a modified version of the meshed IEEE 73 bus Reliability Test System (RTS) [8] with N-1 security. The resulting large (tens of thousands of variables and constraints) nonlinear programs are solved by iteratively adding a number of contingencies selected according to severity, and solving the resulting problem using a warm start based on the result of the previous iteration. This therefore uses the third solution method above, but with an intelligent formulation of the problem.

2 Nomenclature

The various parameters and variables used in the optimisation model are defined here.

2.1 Base case OPF

Sets

B	Set of buses (indexed by b)
L	Set of lines (indexed by l)
G	Set of existing generators (indexed by g)
N	Set of new generators (indexed by n)
X	Set of external sources (indexed by x)

Parameters

$d_b^{(P,Q)}$	(P,Q) demand at bus b
$V_b^{(+,-)}$	(max/min) voltage at b
b_0	Reference/slack bus
$p_g^{(+,-)}$	(max/min) P output of existing generator g
$q_g^{(+,-)}$	(max/min) Q output of g
β_g	Location of G
$s_n^{(+,-)}$	(max/min) capacity of new generator n
β_n	Location of n
ϕ	Power angle of new generators
$p_x^{X,(+,-)}$	(max/min) P supplied by external source x
$q_x^{X,(+,-)}$	(max/min) Q supplied by x
f_l^+	Maximum power flow on line l
$\beta_l^{(1,2)}$	(start,end) bus of l
g_l	Conductance of l
b_l	Susceptance of l
b_l^C	Shunt capacitance of l

Variables

V_b	Voltage at b
δ_b	Voltage angle at b
p_g^{base}	Base P output at b (see later for use)
q_g^{base}	Base Q output at b
p_g	P output of g
q_g	Q output of g
s_n	Capacity of n
p_x^X	P supplied by x
q_x^X	Q supplied by x
$f_l^{(1,2),P}$	P injection onto l at (start,end) bus
$f_l^{(1,2),Q}$	Q injection onto l at (start,end) bus
$f_l^{(1,2),SS}$	Magnitude of S flow on l at (start,end) bus squared
f_l^{bnd}	Upper bound for base/contingency flows on l

2.2 Security model

Sets

M	Set of contingencies considered (indexed by m)
L_m	Set of lines available in contingency m
M^{sm}	Set of contingencies explicitly in security model
M^{nr}	Set of contingencies never removed from security model
M^-	Contingencies removed from security model
M^+	Contingencies added to security model

Parameters

$V_b^{C,(+,-)}$	(max/min) voltage at b in contingency flows
χ	Increase in maximum flows post-contingency

Variables

$V_{m,b}$	Voltage at b in contingency m
$f_{m,l}^{(1,2),P}$	P injections onto l post contingency m
etc.	by analogy with base case

3 Optimisation model

The model for maximizing DG uptake without security constraints (but including fault level constraints) is described in detail in [1]. The objective function in this case is simply the total capacity of the new distributed generators – however, modifying this to perform a cost benefit analysis including the cost of existing generation, or network upgrades, would be relatively straightforward. For transparency, in this paper only lines, loads, DG sites, existing generators and connections to an external transmission network are included. Further features such as VAr sources and tap changing transformers may be added using standard power flow equations without changing the solution technique. Using a continuous variable for DG site capacity is appropriate to a number of distributed sources, where individual generator unit ratings might only be a few MW.

The base case is a standard optimal power flow model apart from the new generator sites. The objective function is the cumulative capacity of all the DG sites, and the new generators are run in constant power factor mode, as is common with DG

For the security model, constraints are added to represent the power flow equations for each contingency considered. In the contingency model, one external source is chosen as the slack bus. Other external sources, DG and load buses are modelled as PQ nodes, while existing generators are PV nodes. Thermal, voltage and generation level constraints are included to ensure that the power flow remains feasible post-contingency. The post-contingency voltage and flow limits may differ from their pre-contingency values.

3.1 OPF without security constraints

Objective function. The goal is to maximise the total capacity of the new distributed generators,

$$\max \sum_{n\in N} s_n \tag{1}$$

Capacity constraint for new generators.

$$s_n^- \le s_n \le s_n^+ \quad \forall\, n \in N \tag{2}$$

Generation level constraint for existing generators.

$$\left.\begin{array}{l} p_g^- \le p_g \le p_g^+ \\ q_g^- \le q_g \le q_g^+ \end{array}\right\} \quad \forall\, g \in G \tag{3}$$

Supply level constraint for external sources.

$$\left.\begin{array}{l} p_x^{X,-} \le p_x^X \le p_x^{X,+} \\ q_x^{X,-} \le q_x^X \le p_x^{X,+} \end{array}\right\} \quad \forall\, x \in X \tag{4}$$

Voltage level constraint.

$$V_b^- \le V_b \le V_b^+ \quad \forall\, b \in B \tag{5}$$

Reference bus. At the reference bus, the voltage angle is zero,

$$\delta_{b_0} = 0 \tag{6}$$

Kirchhof voltage law. At the start bus of lines, $\forall\, l \in L$,

$$\begin{aligned} f_l^{1,P} &= g_l V(\beta_l^1)^2 - \\ &\quad V(\beta_l^1)V(\beta_l^2)\left[g_l \cos\left(\delta(\beta_l^1) - \delta(\beta_l^2)\right)\right. \\ &\quad\quad \left. + b_l \sin\left(\delta(\beta_l^1) - \delta(\beta_l^2)\right)\right] \quad (7) \\ f_l^{1,Q} &= -b_l V(\beta_l^1)^2 - \\ &\quad V(\beta_l^1)V(\beta_l^2)\left[g_l \sin\left(\delta(\beta_l^1) - \delta(\beta_l^2)\right)\right. \\ &\quad\quad \left. - b_l \cos\left(\delta(\beta_l^1) - \delta(\beta_l^2)\right)\right] \quad (8) \end{aligned}$$

At the end bus of lines, $\forall\, l \in L$,

$$\begin{aligned} f_l^{2,P} &= g_l V(\beta_l^2)^2 - \\ &\quad V(\beta_l^2)V(\beta_l^1)\left[g_l \cos\left(\delta(\beta_l^2) - \delta(\beta_l^1)\right)\right. \\ &\quad\quad \left. + b_l \sin\left(\delta(\beta_l^2) - \delta(\beta_l^1)\right)\right] \quad (9) \\ f_l^{2,Q} &= -b_l V(\beta_l^2)^2 - \\ &\quad V(\beta_l^2)V(\beta_l^1)\left[g_l \sin\left(\delta(\beta_l^2) - \delta(\beta_l^1)\right)\right. \\ &\quad\quad \left. - b_l \cos\left(\delta(\beta_l^2) - \delta(\beta_l^1)\right)\right] \quad (10) \end{aligned}$$

Kirchhof current law. Real power conservation. $\forall\, b \in B$,

$$\begin{aligned} &\sum_{g\in G|\beta_g=b} p_g + \sum_{x\in X|\beta_x=b} p_x^X + \sum_{n\in N|\beta_n=b} (\cos\phi) s_n \\ &= \sum_{l\in L|\beta_l^1=b} f_l^{1,P} + \sum_{l\in L|\beta_l^2=b} f_l^{2,P} + d_b^P - p_b^{\text{base}} \quad (11) \end{aligned}$$

Reactive power conservation. $\forall\, b \in B$,

$$\begin{aligned} &\sum_{g\in G|\beta_g=b} q_g + \sum_{x\in X|\beta_x=b} q_x^X + \sum_{n\in N|\beta_n=b} (\sin\phi) s_n \\ &= \sum_{l\in L|\beta_l^1=b} f_l^{1,Q} + \sum_{l\in L|\beta_l^2=b} f_l^{2,Q} + d_b^P - q_b^{\text{base}} \\ &\quad - \frac{(V_b)^2}{2}\left[\sum_{l\in L|\beta_l^1=b} b_l^C + \sum_{l\in L|\beta_l^1=b} b_l^C\right] \quad (12) \end{aligned}$$

Flow constraints at each end of lines.

$$\left.\begin{array}{l} (f_l^{1,P})^2 + (f_l^{1,Q})^2 \le (f_l^{\text{bnd}})^2 \\ (f_l^{2,P})^2 + (f_l^{2,Q})^2 \le (f_l^{\text{bnd}})^2 \end{array}\right\} \quad \forall\, l \in L \tag{13}$$

Lagrange multiplier (LM) constraints. The bus power supply and line capacity Lagrange multipliers come from these constraints. These additional variables are introduced to obtain a single LM given that the flow limits and Kirchhof current law appear in both the base case and contingency flows.

$$f_l^{\text{bnd}} = f_l^+ \quad \forall\, l \in L \tag{14}$$

$$\left.\begin{array}{l} p_b^{\text{base}} = 0 \\ q_b^{\text{base}} = 0 \end{array}\right\} \quad \forall\, b \in B \tag{15}$$

Adding these extra variables carries no significant computational overhead using a standard solver - if it did, an alternative approach is just to add the LMs from the individual constraints. This might be important if the problem structure is exploited, and constraints linking all the contingencies become undesirable.

3.2 Security model

The following constraints are added for all contingencies explicitly included in the security model, i.e. $\forall m \in M^{\text{sm}}$.

Reference/slack bus constraints.

$$\delta_{m,b_0} = 0 \tag{16}$$

$$V_{m,b_0} = V_{b_0} \tag{17}$$

Voltage level constraint

$$V_b^{C,-} \le V_{m,b} \le V_b^{C,+} \quad \forall\, b \in B \tag{18}$$

Supply level constraint for external sources.

$$\left.\begin{array}{l} p_x^{X,-} \le p_{m,x}^X \le p_x^{X,+} \\ q_x^{X,-} \le q_{m,x}^X \le p_x^{X,+} \end{array}\right\} \quad \forall\, x \in X \setminus \{b_0\} \tag{19}$$

Existing generator constraints.

$$\left.\begin{array}{l} V_{m,\beta_g} = V_{\beta_g} \\ q_g^- \le q_{m,g} \le q_g^+ \end{array}\right\} \quad \forall\, g \in G \tag{20}$$

Flow constraints.

$$\left.\begin{array}{l} (f_{m,l}^{1,P})^2 + (f_{m,l}^{1,Q})^2 \le (\chi f_l^{\text{bnd}})^2 \\ (f_{m,l}^{2,P})^2 + (f_{m,l}^{2,Q})^2 \le (\chi f_l^{\text{bnd}})^2 \end{array}\right\} \quad \forall\, l \in L \tag{21}$$

Kirchhof voltage law. These constraints take exactly the same form as before, but for contingency m, a KVL constraint expressing the flows $f_{m,l}^{(1,2)}$ in terms of the contingency voltages $(V_{m,b}, \delta_{m,b})$ is only generated for the available lines $l \in L_m$. $V_{m,b} = 0$ for lines not in L_m.

Kirchhof current law. Real power: $\forall\, b \in B$,

$$\sum_{g \in G|\beta_g = b} p_g + \sum_{x \in X|\beta_x = b} p_{m,x}^X + \sum_{n \in N|\beta_n = b} (\cos\phi) s_n = \sum_{l \in L|\beta_l^1 = b} f_{m,l}^{1,P} + \sum_{l \in L|\beta_l^2 = b} f_{m,l}^{2,P} + d_b^P - p_b^{\text{base}} \quad (22)$$

Reactive power: $\forall\, b \in B$,

$$\sum_{g \in G|\beta_g = b} q_{m,g} + \sum_{x \in X|\beta_x = b} q_{m,x}^X + \sum_{n \in N|\beta_n = b} (\sin\phi) s_n = \sum_{l \in L|\beta_l^1 = b} f_{m,l}^{1,Q} + \sum_{l \in L|\beta_l^2 = b} f_{m,l}^{2,Q} + d_b^P - q_b^{\text{base}} - \frac{(V_{m,b})^2}{2} \left[\sum_{l \in L_m|\beta_l^1 = b} b_l^C + \sum_{l \in L_m|\beta_l^1 = b} b_l^C \right] \quad (23)$$

4 Solution method

4.1 Previous approaches

In a secure DC OPF, it is possible to include individual contingency flow constraints (i.e. to limit the flow on l_1 when l_2 suffers an outage) without including entire contingency power flows in the model due to the linearity of the problem [6]. In the nonlinear AC OPF, however, if one wishes to limit the post-contingency flow on l_1 when l_2 suffers an outage, it is necessary to include the entire post-contingency power flow when an exact solution is required. Many approaches to the solution of large scale AC SCOPFs have therefore involved the pre-selection of a small number of the most significant contingencies; this may involve a sophisticated mathematical selection procedure [9].

The methodology presented here is based on that of Alsac and Stott [10]. They used an iterative process where first the non-secure OPF is solved, and then all contingencies in whose power flows constraint violations occur are added to the security model. The resulting SCOPF is then solved, and the process repeated until no violations are found. While this process was successful for the classical ‘cost-minimisation’ OPF, the ‘DG maximisation’ OPF tends to run the network much harder as the total generation is not restricted by the load (systems are often designed to give low network congestion under minimum cost generator dispatch with N-1 security.) The Stott-Alsac method would thus add too many lines on the early iterations, resulting in large SCOPFs with poorer warm starts. It is therefore more efficient to limit the number of contingencies added on each iteration (in practice, once the worst contingencies are added on the first few iterations, constraint violations disappear in other contingencies not explicitly included.)

4.2 Solution methodology

The solution method for the SCOPF is as follows (see also Fig. 1):

1. Initialise the security model with $M^{\text{sm}} = \emptyset$ (no contingencies initially in security model), $M^{\text{nr}} = M$ (no contingencies yet removed from security model).

2. Solve SCOPF with contingencies M^{sm}. For contingencies which were included in the previous SCOPF, warm start the contingency variables from their values in the previous solution. For contingencies not in the previous OPF, warm start the contingency variables from the previous base case solution.

3. Define $M^- =$ all contingencies in $M^{\text{sm}} \cap M^{\text{nr}}$ with no active voltage, reactive power or flow limit constraints.

4. Run contingency flows for all contingencies in $M \setminus M^{\text{sm}}$, i.e. those not considered in the SCOPF. Define the set M^+ to be the n^+ contingencies whose load flows give the most constraint violations. If fewer than n^+ contingencies give violations, then only include those.

5. If no constraints are violated by these load flows, then the algorithm terminates.

6. Update the security model M^{sm}, adding the contingencies in M^+ and removing those in M^-. Remove those in M^- from $M^{\text{nr}} = M$, the set of contingencies which have never been removed.

7. Go to step 2.

When the algorithm terminates, it does so at a local minimum of the SCOPF including *all* contingencies in M, not just those explicitly included in the security model M^{sm}.

The removal of contingencies in Step 3 attempts to identify those whose explicit consideration is unnecessary – there will usually be a subset of contingencies, whose inclusion in the security model guarantees security in other contingencies. The size of the OPFs solved is therefore reduced, leading to a corresponding reduction in the time taken. The restriction that a contingency may only be removed from the security model once is necessary to ensure eventual termination of the algorithm, as eventually all contingencies must be added if termination has not occured. Without this restriction, the algorithm can cycle, alternately adding and removing the same line. The warm starts using the previous SCOPF solution are vital to the efficiency of the method – solving a large OPF incrementally in this way through successive subproblem solutions is more efficient than making a single direct solution, despite the increased number of solver calls.

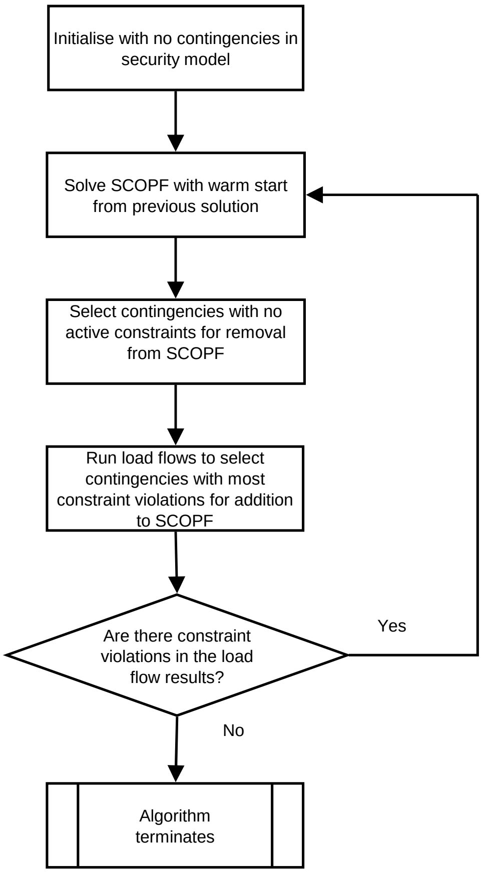

Figure 1: Solution algorithm for the security constrained optimal power flow.

4.3 *Implementation*

The model is coded in the AIMMS optimisation modelling environment, including the load flows (these are performed as constrained optimisation models with a constant objective function.) Use of a modelling language allows for very low development times, and also the deployment of efficient commercial solvers - here the CONOPT generalised reduced gradient solver is used, as it proved to be competitive with the KNITRO interior point solver in terms of speed, and was absolutely reliable in convergence for the SCOPFs in this paper. In contrast, with default settings KNITRO often failed to converge if the number of contingencies considered was not in low single figures. Occasionally, load flows did fail to converge in CONOPT on early iterations of the algorithm. However, as long as these do not occur later on this does not cause any risk of the method terminating unsuccessfully.

5 Secure OPF results

5.1 *Test problem*

To demonstrate the solution method, a modified version of the IEEE 73-bus Reliability Test System (RTS) is used – the layout is shown in Figure 3 and circuit parameters may be found in [8]. This consists of three identical 24-bus networks, and interconnections between them. The RTS is not intended to be representative of any particular power system, but it provides a convenient reasonably large scale network on which to demonstrate the SCOPF solution method. While the network used here is almost fully meshed with N-1 security, the method is generic and can be applied equally well to other topologies (e.g. radial networks with reconfiguration-type security models as well as N-k security) and voltage levels.

All generators from the original RTS are removed. DG units of up to 1 GW capacity are allowed to be connected at buses 101, 102, 107, 115, 122, 201, 202, 207, 215, 222, 301, 302, 307, 315 and 322. Existing conventional generators with real power output 800 MW and reactive power output between $\pm$400 MVAr are sited at buses 118, 218 and 318. External sources of unlimited capacity are placed at buses 123, 223, 323 and 325 (the latter being the reference/slack bus.) In order to ensure feasibility of the SCOPF model, all real and reactive power demands take half their value from the original RTS – the total demand is then 4.22 GW. Line resistances, reactances and capacitances are unaltered. The 105 contingencies considered are single-line outages of all lines except lines 10, 11, 16, 17 and 23 in each region (211 and 311 are radial lines leading to a single load/generator bus, and including the others also leaves an infeasible problem with N-1 security.)

5.2 *Results*

The time taken for completion of the algorithm is plotted in Figure 2 against the maximum number of contingencies added per iteration n^+. In all cases, the locally optimal objective function value found (i.e. the total DG capacity) was 3.08 GW. Without security constraints the value was 4.58 GW.

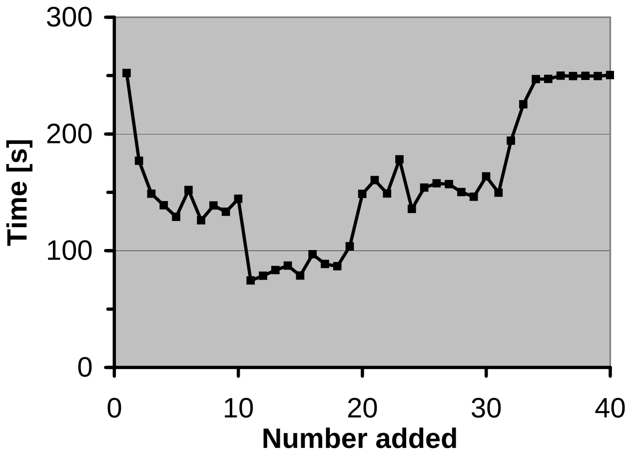

Figure 2: Time taken to solve to local optimality, for a range of numbers of contingencies added in each step (n^+ in the algorithm description.)

For comparison, the code takes around 11300 seconds to run when a direct solution of the problem with all contingencies is attempted from a flat start (all variables zero except the voltage levels, which are set to 1 p.u.), the resulting mathematical program having around 100,000 variables and constraints. Using a non-secure OPF solution to warm start a direct solution including all contingencies took 513 s. The base case gives 1228 constraints and 1013 variables, and each contingency adds 1269 constraints and 877 variables.

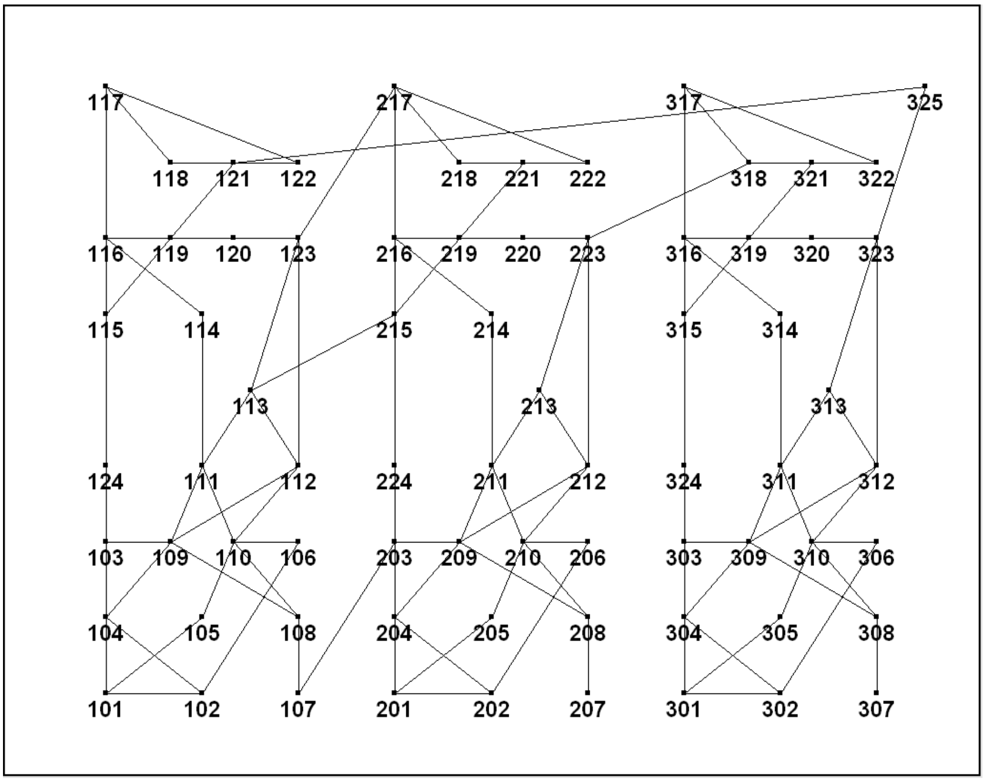

Figure 3: Layout of the IEEE 73 bus reliability test system. For full details and network parameters see [8].

Each data point in the smoothed data set is a simple mean of the value at that number of contingencies added n^+, and its two neighbours. As timings of runs in AIMMS (running under Windows XP with an Intel 2.13 GHz dual core processor) varied according to what other processes were active, the times given are the *smallest* of three runs, to reflect most accurately the actual processor time used. The calculation results were repeatable between runs.

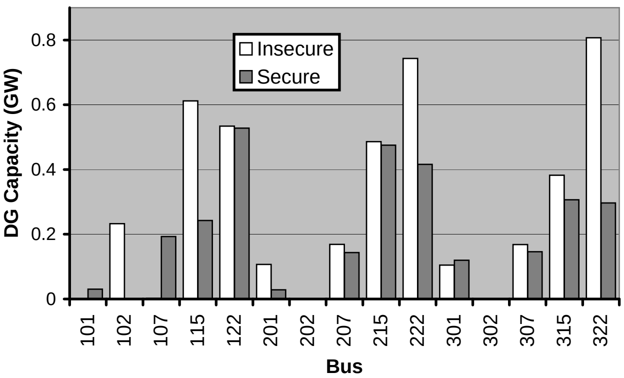

Figure 4: Optimal DG site capacities with and without security constraints (the secure results use $n^+ = 29$, which gives the highest total capacity.) The generator capacities do not all scale by the same factor when security constraints are introduced.

The smallest run times occur adding a maximum of $n^+ = 11$ to 19 contingencies on each iteration. The total time taken is a trade-off between the time for each OPF solution (smaller at low n^+ due to both the smaller optimisation problems and the better warm starts) and the number of iterations (which as expected is smaller at large n^+.) The lower end of this range seems to be a good choice for other problems solved on the same network, as there is an unknown level of risk that if n^+ is made too large then a large, hard mathematical program may be encountered despite the use of warm starts.

All run times are approximately the same for $n^+ \geq 34$. This is because the non-secure result violates constraints in 34 contingencies – as n^+ is the maximum number of contingencies which can be added on any iteration, and the number of violations decreases when security constraints are introduced, the same sequence of problems is solved in each of these cases. This is actually the Alsac-Stott method, where *all* contingencies with violations are added on each iteration – the approach of limiting the number added is thus more efficient in this case.

The individual optimal DG site capacities with and without security constraints are compared in Figure 4. When N-1 security is introduced, the capacities do not scale equally – for instance, the capacity at bus 322 decreases by 63%, while that at bus 122 barely changes.

5.3 *Convexity issues*

The solution found using the 'sequential warm start' is necessarily a local solution of the SCOPF including all contingencies – indeed, this is confirmed by direct solution of the problem. As only one local solution was found, it seems likely that this is the global maximum. However, there is no way of proving this, as for non-convex nonlinear optimisation models such as this there are no efficient general purpose global solution methods available.

In practice therefore, it is typically best to perform multiple runs of the same problem with different starting points. In a sense, this has already been done, and only one local solution was found. If multiple starts of the same process are required, this is complicated by the criticality of warm starts to the efficiency of the algorithm (possibly rendering impractical the simplest option of choosing random starts with initial values for variables chosen from their whole range.)

It should be noted that in planning applications such as this, finding the global optimum is not critical – the technique is valuable if it finds an investment plan which is better than those obtained by other means, and any good local optimum may therefore suffice. This might not be the case however where an AC OPF is given statutory authority, e.g. if it is used for generator dispatch in a pool system, where similar objective functions may give widely differing generation schedules and power prices. It is clearly desirable in this case for the result found to be independent of the start point, for different people using the same mathematical analysis of the situation to be guaranteed the same result, and for that result to be the true global optimum if at all possible.

Despite its lower level of engineering detail, one major advantage of the linearised DC OPF is its convexity, which guarantees that any local optimum is globally optimal – this ensures that the criteria listed in the previous paragraph are met.

6 Conclusion

A method has been presented for using an optimal power flow to maximise distributed generation uptake in a power network where post-contingency security constraints are imposed. The motivation behind this model is to assess the network's capacity for new generation. An efficient solution technique is used for the resulting optimisation problem, based on repeatedly adding a limited selection of contingencies in which power flow constraints are violated, together with warm starts from earlier solutions. These were both significant in reducing run times – a direct solution including all contingencies took significantly longer, as did adding all contingencies with violations at each stage. The algorithm has been tested on an adapted IEEE 73-bus reliability test system, but it is generic and may be used for a full range of network topologies, voltage levels and post-contingency security models.

It is important to note that as with any large non-convex NLP, it is impractical to find a solution which guarantees global optimality. In this case, only one local solution was found. This is therefore likely to be the global solution, but possibilities are discussed for using multiple start techniques for finding good solutions where more than one local solution exists.

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